EDU QUATE

Quotes Taken From: *(34 in doc as of 9/13/18)*

- *The Myth of "I'm Bad at Math,"* The Atlantic, 2013
- *Why do Americans Stink at Math?,* The New York Times Magazine, 2014
- *Teaching Math in the 21st Century* , Deborah Lowenberg Ball
- *A Mathematician's Lament*, Paul Lockhart
- Learning that Lasts, Ron Berger, Libby Woodfin, Anne Vilen, 2016
- Mathematical Mindsets, Jo Boaler, 2016
- Principles to Actions: Ensuring Mathematical Success for All, NCTM, 2014

"Improving the mathematics learning of every child depends on making central the learning opportunities of our teachers."

- from Deborah Loewenberg Ball, Mathematics in the 21st Century: What Mathematical Knowledge is Needed for Teaching Mathematics?

" …teachers – like all other adults in this country – are graduates of the system we seek to improve."

- Deborah Loewenberg Ball in Mathematics in the 21st Century: What Mathematical Knowledge is Needed for Teaching Mathematics?

More than half of fourth graders taking the 2013 National Assessment of Educational Progress could not accurately read the temperature on a neatly drawn thermometer. (They did not understand that each hash mark represented two degrees rather than one, leading many students to mistake 46 degrees for 43 degrees.) … Even in Massachusetts, one of the country’s highest-performing states, math students are more than two years behind their counterparts in Shanghai.

A 2012 study comparing 16-to-65-year-olds in 20 countries found that Americans rank in the bottom five in numeracy. On a scale of 1 to 5, 29 percent of them scored at Level 1 or below, meaning they could do basic arithmetic but not computations requiring two or more steps. One study that examined medical prescriptions gone awry found that 17 percent of errors were caused by math mistakes on the part of doctors or pharmacists. A survey found that three-quarters of doctors inaccurately estimated the rates of death and major complications associated with common medical procedures, even in their own specialty areas.

One of the most vivid arithmetic failings displayed by Americans occurred in the early 1980s, when the A&W restaurant chain released a new hamburger to rival the McDonald’s Quarter Pounder. With a third-pound of beef, the A&W burger had more meat than the Quarter Pounder; in taste tests, customers preferred A&W’s burger. And it was less expensive. A lavish A&W television and radio marketing campaign cited these benefits. Yet instead of leaping at the great value, customers snubbed it. Only when the company held customer focus groups did it become clear why. The Third Pounder presented the American public with a test in fractions. And we failed. Misunderstanding the value of one-third, customers believed they were being overcharged. Why, they asked the researchers, should they pay the same amount for a third of a pound of meat as they did for a quarter-pound of meat at McDonald’s. The “4” in “¼,” larger than the “3” in “⅓,” led them astray.

Students learn not math but, in the words of one math educator, answer-getting. Instead of trying to convey, say, the essence of what it means to subtract fractions, teachers tell students to draw butterflies and multiply along the diagonal wings, add the antennas and finally reduce and simplify as needed. The answer-getting strategies may serve them well for a class period of practice problems, but after a week, they forget.

Teachers primarily learn to teach by recalling their memories of having been taught, about 13,000 hours of instruction during a typical childhood — a problem since their instruction wasn't very good.

Almost every school in America is filled with teachers and students who declare, without apparent shame, "I am just not a math person," or say, "I just don't get math; I've never been good at math." What teacher would glibly say, "I'm just not a reading person" or "I just don't get writing?" Somehow we have decided that it is acceptable to be "bad at math" – even if you are a teacher. That culture needs to change, especially in schools.

The first and most important change for a school to make to improve mathematical achievement is a shift in the adult professional culture in the building toward one that embraces mathematical learning. This takes a willingness of the faculty to address their own mathematical strengths and weaknesses and to commit to improving their own mathematical content knowledge – which goes hand in hand with learning new teaching strategies. It means that faculty members must marshal their mathematical courage to take risks together, grapple with problems together, and get excited about learning mathematics together.

"Challenge is at the heart of deeper instruction. Grappling with new ideas about problems will productively challenge students when they have enough background knowledge to feel anchored, enough scaffolding to feel supported, and enough time and intellectual freedom to wrestle with complex ideas that stimulate thinking. A productive challenge stretches students to go beyond what they may think is possible. This stretch leads to new learning."

Because noncognitive qualities like grit, curiosity, self-control, optimism, and conscientiousness are often described, with some accuracy, as skills, educators eager to develop these qualities in their students quite naturally tend to treat them like the skills that we already know how to teach: reading, calculating, analyzing, and so on. And as the value of noncognitive skills has become more widely acknowledge, demand has grown for a curriculum or textbook or a teaching strategy to guide us in helping students develop these skills…In practice though, it hasn't been so simple…Many of the educators I encountered who seemed best able to engender noncognitive abilities in their students never said a word about these skills in the classroom.

When students are learning with understanding, the learning process becomes a genuine partnership between the students and teacher and among the students. The students feel empowered to share their ideas, ask questions, and contribute problems for the class to explore rather than sit back and be told what to do.

"It is my job as their teacher to establish routines and provide problems … but when students begin to assume responsibility for questioning and probing, then I know they are learning with understanding and, more important, they know they are capable of doing so and they know what to do to build that understanding. Learning math to them no longer means relying on the teacher to tell them what to do, but rather it is dependent on their strategies, observations, questions, and explanations."

"Mathematics is an art. The difference between math and the other arts, such as music and painting, is that our culture does not recognize it as such. Everyone understands that poets, painters, and musicians create works of art, and are expressing themselves in word, image, and sound. In fact, our society is rather generous when it comes to creative expression;

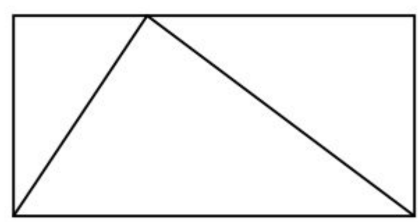
architects, chefs, and even television directors are considered to be working artists. So why not mathematicians?"

I was surprised to find myself in a regular school classroom— no easels, no tubes of paint. "Oh we don't actually apply paint until high school," I was told by the students. "In seventh grade we mostly study colors and applicators." They showed me a worksheet. On one side were swatches of color with blank spaces next to them. They were told to write in the names. "I like painting," one of them remarked, "they tell me what to do and I do it. It's easy!" After class I spoke with the teacher. "So your students don't actually do any painting?" I asked. "Well, next year they take Pre-Paint-by-Numbers. That prepares them for the main Paint-by-Numbers sequence in high school. So they'll get to use what they've learned here and apply it to real-life painting situations— dipping the brush into paint, wiping it off, stuff like that. Of course we track our students by ability. The really excellent painters— the ones who know their colors and brushes backwards and forwards— they get to the actual painting a little sooner, and some of them even take the Advanced Placement classes for college credit. But mostly we're just trying to give these kids a good foundation in what painting is all about, so when they get out there in the real world and paint their kitchen they don't make a total mess of it." "Um, these high school classes you mentioned…" "You mean Paint-by-Numbers? We're seeing much higher enrollments lately. I think it's mostly coming from parents wanting to make sure their kid gets into a good college. Nothing looks better than Advanced Paint-by-Numbers on a high school transcript." "Why do colleges care if you can fill in numbered regions with the corresponding color?" "Oh, well, you know, it shows clear-headed logical thinking. And of course if a student is planning to major in one of the visual sciences, like fashion or interior decorating, then it's really a good idea to get your painting requirements out of the way in high school." "I see. And when do students get to paint freely, on a blank canvas?" "You sound like one of my professors! They were always going on about expressing yourself and your feelings and things like that—really way-out-there abstract stuff. I've got a degree in Painting myself, but I've never really worked much with blank canvases. I just use the Paint-by-Numbers kits supplied by the school board."

"Nobody has the faintest idea what it is that mathematicians do. The common perception seems to be that mathematicians are somehow connected with science— perhaps they help the scientists with their formulas, or feed big numbers into computers for some reason or other. There is no question that if the world had to be divided into the "poetic dreamers" and the "rational thinkers" most people would place mathematicians in the latter category."

There is nothing as dreamy and poetic, nothing as radical, subversive, and psychedelic, as mathematics. It is every bit as mind blowing as cosmology or physics (mathematicians conceived of black holes long before astronomers actually found any), and allows more freedom of expression than poetry, art, or music (which depend heavily on properties of the physical universe). Mathematics is the purest of the arts, as well as the most misunderstood.

A mathematician, like a painter or poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas. So mathematicians sit around making patterns of ideas. What sort of patterns? What sort of ideas? Ideas about the rhinoceros? No, those we leave to the biologists. Ideas about language and culture? No, not usually. These things are all far too complicated for most mathematicians' taste. If there is anything like a unifying aesthetic principle in mathematics, it is this: simple is beautiful. Mathematicians enjoy thinking about the simplest possible things, and the simplest possible things are imaginary.



if I'm in the mood to think about shapes— and I often am— I might imagine a triangle inside a rectangular box. I wonder how much of the box the triangle takes up? Two-thirds maybe? The important thing to understand is that I'm not talking about this drawing of a triangle in a box. Nor am I talking about some metal triangle forming part of a girder system for a bridge. There's no ulterior practical purpose here. I'm just playing. That's what math is— wondering, playing, amusing yourself with your imagination… This is a major theme in mathematics: things are what you want them to be. You have endless choices; there is no reality to get in your way. On the other hand, once you have made your choices then your new creations do what they do, whether you like it or not. This is the amazing thing about making imaginary patterns: they talk back! The triangle takes up a certain amount of its box, and I don't have any control over what that amount is. There is a number out there, maybe it's two-thirds, maybe it isn't, but I don't get to say what it is. I have to find out what it is.

The cultural problem is a self-perpetuating monster: students learn about math from their teachers, and teachers learn about it from their teachers, so this lack of understanding and appreciation for mathematics in our culture replicates itself indefinitely. Worse, the perpetuation of this "pseudo-mathematics," this emphasis on the accurate yet mindless manipulation of symbols, creates its own culture and its own set of values. Those who have become adept at it derive a great deal of self-esteem from their success. The last thing they want to hear is that math is really about raw creativity and aesthetic sensitivity. Many a graduate student has come to grief when they discover, after a decade of being told they were "good at math," that in fact they have no real mathematical talent and are just very good at following directions. Math is not about following directions, it's about making new directions.

At no time [in school] are students let in on the secret that mathematics, like any literature, is created by human beings for their own amusement; that works of mathematics are subject to critical appraisal; that one can have and develop mathematical taste. A piece of mathematics is like a poem, and we can ask if it satisfies our aesthetic criteria: Is this argument sound? Does it make sense? Is it simple and elegant? Does it get me closer to the heart of the matter? Of course there's no criticism going on in school— there's no art being done to criticize!

Why don't we want our children to learn to do mathematics? Is it that we don't trust them, that we think it's too hard? We seem to feel that they are capable of making arguments and coming to their own conclusions about Napoleon, why not about triangles? I think it's simply that we as a culture don't know what mathematics is. The impression we are given is of something very cold and highly technical, that no one could possibly understand— a self fulfilling prophecy if there ever was one.

Attempts to present mathematics as relevant to daily life inevitably appear forced and contrived: "You see kids, if you know algebra then you can figure out how old Maria is if we know that she is two years older than twice her age seven years ago!" (As if anyone would ever have access to that ridiculous kind of information, and not her age.) Algebra is not about daily life, it's about numbers and symmetry— and this is a valid pursuit in and of itself:

> Suppose I am given the sum and difference of two numbers. How can I figure out what the numbers are themselves?

Here is a simple and elegant question, and it requires no effort to be made appealing. The ancient Babylonians enjoyed working on such problems, and so do our students. (And I hope T 9 you will enjoy thinking about it too!) We don't need to bend over backwards to give mathematics relevance. It has relevance in the same way that any art does: that of being a meaningful human experience.

The main problem with school mathematics is that there are no problems. Oh, I know what passes for problems in math classes, these insipid "exercises." "Here is a type of problem. Here is how to solve it. Yes it will be on the test. Do exercises 1-35 odd for homework." What a sad way to learn mathematics: to be a trained chimpanzee.

But a problem, a genuine honest-to-goodness natural human question— that's another thing. How long is the diagonal of a cube? Do prime numbers keep going on forever? Is infinity a number? How many ways can I symmetrically tile a surface? The history of mathematics is the history of mankind's engagement with questions like these, not the mindless regurgitation of formulas and algorithms (together with contrived exercises designed to make use of them).

I can understand the idea of training students to master certain techniques— I do that too. But not as an end in itself. Technique in mathematics, as in any art, should be learned in context. The great problems, their history, the creative process— that is the proper setting.

Give your students a good problem, let them struggle and get frustrated. See what they come up with. Wait until they are dying for an idea, then give them some technique. But not too much.

If teaching is reduced to mere data transmission, if there is no sharing of excitement and wonder, if teachers themselves are passive recipients of information and not creators of new ideas, what hope is there for their students? If adding fractions is to the teacher an arbitrary set of rules, and not the outcome of a creative process and the result of aesthetic choices and desires, then of course it will feel that way to the poor students. Teaching is not about information. It's about having an honest intellectual relationship with your students. It requires no method, no tools, and no training. Just the ability to be real. And if you can't be real, then you have no right to inflict yourself upon innocent children.

You can't teach teaching. Schools of education are a complete crock. Oh, you can take classes in early childhood development and whatnot, and you can be trained to use a blackboard "effectively" and to prepare an organized "lesson plan" (which, by the way, insures that your lesson will be planned, and therefore false), but you will never be a real teacher if you are unwilling to be a real person. Teaching means openness and honesty, an ability to share excitement, and a love of learning. Without these, all the education degrees in the world won't help you, and with them they are completely unnecessary.

School mathematics concerns itself with many things that have nothing to do with the ability to get along in society— algebra and trigonometry, for instance. These studies are utterly irrelevant to daily life. … if we are going to include such things as part of most students' basic education, that we do it in an organic and natural way. … just because a subject happens to have some mundane practical use does not mean that we have to make that use the focus of our teaching and learning. It may be true that you have to be able to read in order to fill out forms at the DMV, but that's not why we teach children to read. We teach them to read for the higher purpose of allowing them access to beautiful and meaningful ideas. Not only would it be cruel to teach reading in such a way— to force third graders to fill out purchase orders and tax forms— it wouldn't work! We learn things because they interest us now, not because they might be useful later. But this is exactly what we are asking children to do with math.

"Before you can write your own poems you need to learn the alphabet. The process has to begin somewhere. You have to walk before you can run."

No! You have to have something you want to run toward. Children can write poems and stories as they learn to read and write. A piece of writing by a six-year-old is a wonderful thing, and the spelling and punctuation errors don't make it less so. Even very young children can invent songs, and they haven't a clue what key it is in or what type of meter they are using.

"But isn't math different? Isn't math a language of its own, with all sorts of symbols that have to be learned before you can use it?"

Not at all. Mathematics is not a language, it's an adventure. Do musicians "speak another language" simply because they choose to abbreviate their ideas with little black dots? If so, it's no obstacle to the toddler and her song. Yes, a certain amount of mathematical shorthand has evolved over the centuries, but it is in no way essential. Most mathematics is done with a friend over a cup of coffee, with a diagram scribbled on a napkin. Mathematics is and always has been about ideas, and a valuable idea transcends the symbols with which you choose to represent it. As Gauss once remarked, "What we need are notions, not notations."

Isn't one of the purposes of mathematics education to help students think in a more precise and logical way, and to develop their "quantitative reasoning skills?" Don't all of these definitions and formulas sharpen the minds of our students?

No they don't. If anything, the current system has the opposite effect of dulling the mind. Mental acuity of any kind comes from solving problems yourself, not from being told how to solve them.

What about students who are interested in pursuing a career in science or engineering? Don't they need the training that the traditional curriculum provides? Isn't that why we teach mathematics in school?

How many students taking literature classes will one day be writers? That is not why we teach literature, nor why students take it. We teach to enlighten everyone, not to train only the future professionals. In any case, the most valuable skill for a scientist or engineer is being able to think creatively and independently. The last thing anyone needs is to be trained.

The "ladder myth"— is the idea that mathematics can be arranged as a sequence of "subjects" each being in some way more advanced, or "higher" than the previous. The effect is to make school mathematics into a race— some students are "ahead" of others, and parents worry that their child is "falling behind." And where exactly does this race lead? What is waiting at the finish line? It's a sad race to nowhere. In the end you've been cheated out of a mathematical education, and you don't even know it.

Real mathematics doesn't come in a can— there is no such thing as an Algebra II idea. Problems lead you to where they take you. Art is not a race. The ladder myth is a false image of the subject, and a teacher's own path through the standard curriculum reinforces this myth and prevents him or her from seeing mathematics as an organic whole. As a result, we have a math curriculum with no historical perspective or thematic coherence, a fragmented collection of assorted topics and techniques, united only by the ease in which they can be reduced to step-bystep procedures.

Another example is the training of students to express information in an unnecessarily complicated form, merely because at some distant future period it will have meaning. Does any middle school algebra teacher have the slightest clue why he is asking his students to rephrase “the number x lies between three and seven” as $|x - 5| < 2$? Do these hopelessly inept textbook authors really believe they are helping students by preparing them for a possible day, years hence, when they might be operating within the context of a higher-dimensional geometry or an abstract metric space? I doubt it. I expect they are simply copying each other decade after decade, maybe changing the fonts or the highlight colors, and beaming with pride when an school system adopts their book, and becomes their unwitting accomplice.

Mathematics is about problems, and problems must be made the focus of a students mathematical life. Painful and creatively frustrating as it may be, students and their teachers should at all times be engaged in the process— having ideas, not having ideas, discovering patterns, making conjectures, constructing examples and counterexamples, devising arguments, and critiquing each other’s work. Specific techniques and methods will arise naturally out of this process, as they did historically: not isolated from, but organically connected to, and as an outgrowth of, their problem-background.

English teachers know that spelling and pronunciation are best learned in a context of reading and writing. History teachers know that names and dates are uninteresting when removed from the unfolding backstory of events. Why does mathematics education remain stuck in the nineteenth century? Compare your own experience of learning algebra with Bertrand Russell’s recollection:

> “I was made to learn by heart: ‘The square of the sum of two numbers is equal to the sum of their squares increased by twice their product.’ I had not the vaguest idea what this meant and when I could not remember the

words, my tutor threw the book at my head, which did not stimulate my intellect in any way."

Are things really any different today?

Bob Moses, Civil Rights Activist and Founder of The Algebra Project "Today, I want to argue, the most urgent social issue affecting poor people and people of color is economic access. In today's world, economic access and full citizenship depend crucially on math and science literacy." (Moses, Bob, "Radical Equations: Math Literacy and Civil Rights", pg. 5)

Jean Anyon, Professor of Urban Education at the Graduate Center of the City University of New York "I believe it is important for educators, public policy analysts, and practitioners to take hold of the fact that economic policies yield widespread low-wage work even among an increasingly educated workforce… Unless we make some changes in the way the macro-economy works, economic policy will trump not only urban school reform, but individual educational achievement of urban students as well." (Anyon, Jean, "Radical Possibilities: Public Policy, Urban Education, and a New Social Movement", pg. 29)

Ubiratan D'Ambrosio, Brazilian Educator and 'Father' of Ethnomathematics "It is important to show students the presence of math in a world of techno-science, but also to reassure students of their culture roots and show them there is a dynamic in the evolution of mathematical knowledge to which all people – not just the "heroes" - are recognized in the traditional histories of mathematics." (D'Ambrosio, Ubiratan, "Mathematics as Crucial and Timely for Shaping a New Civilization," Talk at NCTM National Conference, April 2006)

Paulo Freire, world-renowned education scholar and author "The educators of this country have much besides content to teach to boys and girls, no matter from what social class they come. They have much to teach through the example of fighting for the

> fundamental changes we need, of fighting against authoritarianism and for democracy… Our job requires dedication to overcoming social injustice. ”(Freire, Paulo, “Teachers as Cultural Workers: Lessons to Those Who Dare Teach”, pg. 58)

Topics not taught in the classroom through traditional mathematics curriculums:

Political, Economic, and Social Issues:

- Prisons, racial profiling, the death penalty
- Poverty, minimum vs. living wage, sweatshops
- Housing, gentrification, homeownership
- War, defense budgets, military recruiting
- Public health, AIDS, asthma, health insurance
- Educational funding and equity, high stakes testing, class size
- Environmental racism, pollution, resource availability

Financial Education

- Credit cards, managing debt, paying for college
- Saving/budgeting money, opening bank accounts
- High-cost loans (rent-to-own stores, check cashers, refund-anticipatory loans, payday, etc.)
- Filing taxes
- Remittance rates